How to write mathematics

Brendon Anderson

8 July 2019

Here are my take-away notes from the essay "How to write mathematics" by Paul R. Halmos:

1. *Have something to say.* Have a clearly defined subject. Don't have no ideas, and don't have too many. This is the most important ingredient in writing.

2. *Speak to someone.* Have an idea of the vague audience you aim to hit, but also keep a specific person in mind. Anticipate their difficulties, and write accordingly.

3. *Organize.* Make an outline before writing, the graph of which should be a tree with logical titling, arrangement, and flow. Do not include too much, and do not leave out too much.

4. *Think about notation.* Take a few hours to list out the variables and symbols going to be used. Ensure no clashes. Use harmonious symbols, e.g., $a_1x_1 + a_2x_2$ rather than $ax_1 + bx_2$. Do not freeze certain symbols into specific uses, e.g. writing "$\dots (x, y, z)$-space$\dots$".

5. *Write in spirals.* Write in the following order: Section 1, Section 2, Section 1, Section 2, Section 3, Section 1, and so on. Make the first draft of each section rough and quick. On the second draft of each section, don't edit, *write again*, from scratch. After the first draft of the document is complete, rewrite it again and add preface, preliminaries, table of contents, appendices, etc. The third rewrite should be minimal. Send third draft to others for review.

6. *Organize in spirals.* While writing in spirals, consider the organization of material. When rewriting earlier sections, take into account the concepts you have written about in the later sections, so as to guide and foreshadow ideas, examples, and terminology.

7. *Use good English.* Use correct English. Know the differences between "that" and "which", "any" and "all", etc. Don't be pedantic, but be correct and precise.

8. *Be honest and open.* The goal of the author should be to make the reading smooth and gain the reader's confidence. Do not state something is obvious unless it is. If an example will come later, state that. If you won't prove something, give a reference.

9. *Leave out the irrelevant.* Emphasize the results, not the details surrounding the result. For proofs, give introductory comments and motivation, then the statement, then the proof. The reader will follow the details of a mathematical theorem better if they already know what the theorem is supposed to say! Try to keep theorem statements to a short single sentence.

10. *Use repetition for good.* Use the same wording, in related theorems for instance, to bring the reader familiarity. When doing so, explicitly point out the difference that the second instance carries from the first. However, do not repeat statements twice using different wording in order to drive the point home. This can cause confusion. Additionally, do not repeat proof techniques or steps. Instead, write a lemma which can be called upon multiple times.

11. *Personal pronouns are okay.* Just try to remain smooth and unobtrusive. Use factual, declarative statements as much as possible, but don't avoid pronouns if it makes things sound awkward. Use imperatives: "To find p, multiply q by r." Use "we" to mean the author and the reader. Try to avoid "we" if it means the authors, since it leaves the reader out.

12. *Use correct words.* For instance, "any" is ambiguous. For example, "Prove any number has unit magnitude." In this case, "any" might mean "prove one number with unit magnitude", or "prove all numbers have unit magnitude." Replace it with "each" or "every"/"all". Avoid "where" as a lazy afterthought; define things first without resorting to "where".

13. *Use technical terms correctly.* A function is f, not $f(x)$. ($f(x)$ is the value of f evaluated at x.) Sequence means "function with domain of natural numbers." Use "contain" for $\in$ and "include" for $\subset$. More generally, use consistent terminology-notation associations. In general, avoid technical terms.

14. *Resist symbols.* Try to write as if you were explaining the concepts verbally. Do not use symbols if they are "free" (unbound to other equations or symbols). Do not use references and labels, e.g., "Equation (42)", unless necessary.

15. *Use symbols correctly.* The symbols $\in$, $\subset$, and $<$ are read "is in", "is included", and "is less than", not "in", "included", and "less than". We wouldn't ever say "1 is < 3." Don't start a sentence with a symbol. Don't put a symbol directly after a comma. Break up prose.

16. *Stop.* Write the damn exposition and let it sit. Don't continue chasing perfection or additions, as such goals are neverending.