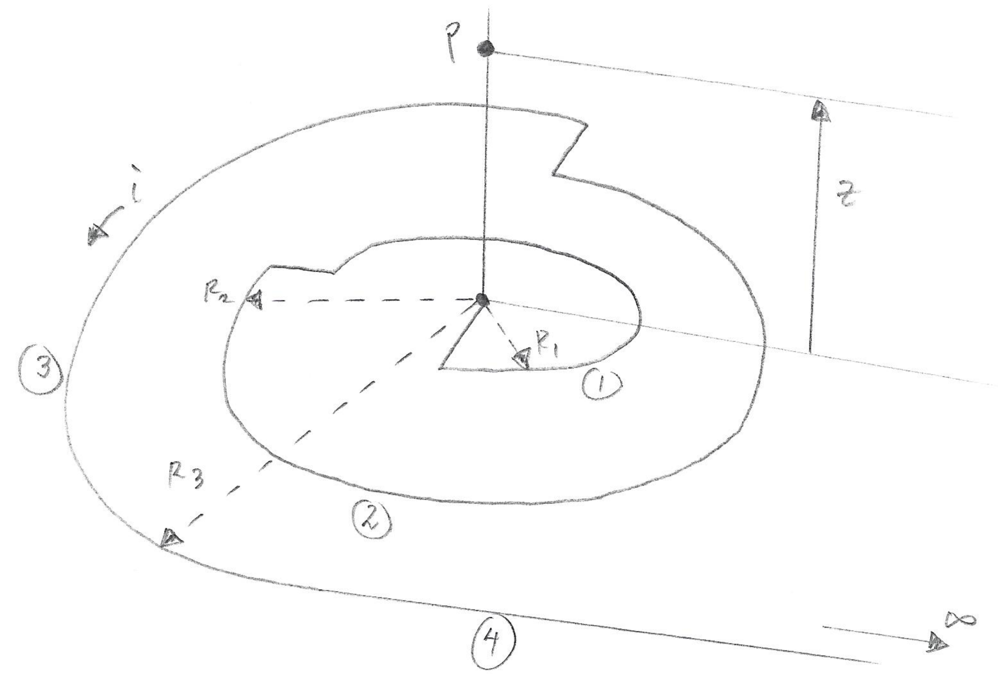
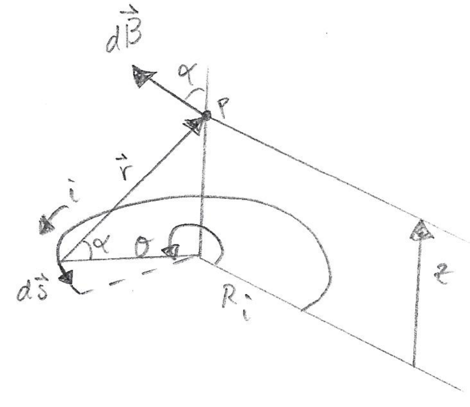
#1:





$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}$$

$$dB_z = \frac{\mu_0 i}{4\pi}\frac{\overset{R_i\,d\theta}{\cancel{ds\,r}}}{r^{\cancel{3}\,2}}\cos\alpha$$

$$r = (R_i^2+z^2)^{1/2}$$

$$\cos\alpha = \frac{R_i}{(R_i^2+z^2)^{1/2}}$$

$$dB_z = \frac{\mu_0 i}{4\pi}\frac{R_i^2}{(R_i^2+z^2)^{3/2}}\,d\theta$$

$$\therefore\ B_z = \frac{\mu_0 i}{4\pi}\frac{R_i^2\,\theta}{(R_i^2+z^2)^{3/2}}$$

for an arbitrary arc length.

$$\therefore \quad \vec{B}_1 = \frac{\mu_0 i R_1^2 \cdot \frac{3}{2}\cancel{\pi}}{4\cancel{\pi}(R_1^2+z^2)^{3/2}}\hat{k}$$

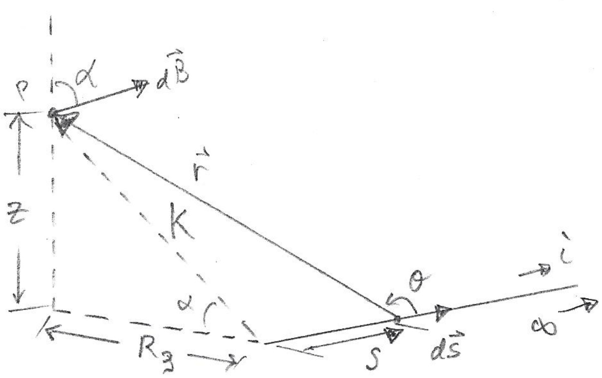
$$\vec{B}_1 = \frac{3\mu_0 i R_1^2}{8(R_1^2+z^2)^{3/2}}\hat{k} \quad \text{———} \quad (1)$$

$$\vec{B}_2 = \frac{\mu_0 i R_2^2 \, \frac{3}{2}\cancel{\pi}}{4\cancel{\pi}(R_2^2+z^2)^{3/2}}\hat{k}$$

$$\vec{B}_2 = \frac{3\mu_0 i R_2^2}{8(R_2^2+z^2)^{3/2}}\hat{k} \quad \text{———} \quad (2)$$

$$\vec{B}_3 = \frac{\mu_0 i R_3^2 \cdot \cancel{\pi}}{4\cancel{\pi}(R_3^2+z^2)^{3/2}}\hat{k}$$

$$\vec{B}_3 = \frac{\mu_0 i R_3^2}{4(R_3^2+z^2)^{3/2}}\hat{k} \quad \text{———} \quad (3)$$



$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}$$

$$dB_z = \frac{\mu_0 i}{4\pi}\frac{ds\,\cancel{r\sin\theta}\,K}{r^3}\cos\alpha$$

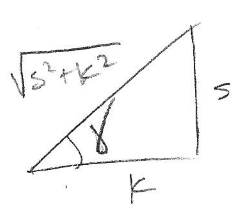
$$r = (s^2+K^2)^{1/2}$$

$$\cos\alpha = \frac{R_3}{K}$$

$$dB_z = \frac{\mu_0 i}{4\pi}\frac{R_3\,ds}{(s^2+K^2)^{3/2}}$$

let $s = K\tan\gamma$

$ds = K\sec^2\gamma\,d\gamma$



$$dB_z = \frac{\mu_0 i R_3}{4\pi}\frac{K\sec^2\gamma\,d\gamma}{K^3\sec^3\gamma}$$

$$\int dB_z = \frac{\mu_0 i R_3}{4\pi K^2}\int \cos\gamma\,d\gamma$$

$$\therefore\ B_z = \frac{\mu_0 i R_3}{4\pi K^2}\ \sin\gamma \Big|$$

$$\sin\gamma = \frac{s}{\sqrt{s^2+K^2}}$$

$$B_z = \frac{\mu_0 i R_3}{4\pi K^2}\ \frac{s}{\sqrt{s^2+K^2}}\Bigg|_{s=0}^{\infty}$$

$$B_z = \frac{\mu_0 i R_3}{4\pi K^2}\left[1-0\right]$$

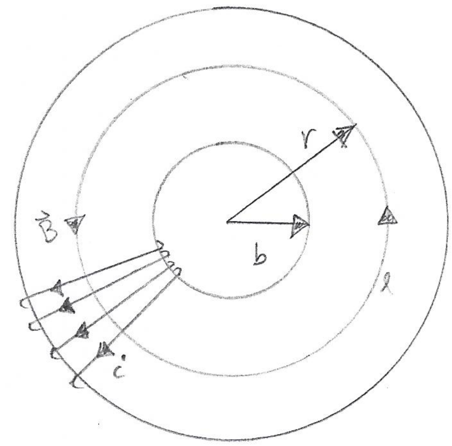
$$K^2 = z^2+R_3^2$$

$$\therefore\ \vec{B}_4 = \frac{\mu_0 i R_3}{4\pi\left(z^2+R_3^2\right)}\ \hat{k} \text{ ——————— } (4)$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 \longleftarrow (1)(2)(3)(4)$$

$$\boxed{\vec{B} = \left[\frac{3\mu_0 i R_1^2}{8\left(R_1^2+z^2\right)^{3/2}} + \frac{3\mu_0 i R_2^2}{8\left(R_2^2+z^2\right)^{3/2}} + \frac{\mu_0 i R_3^2}{4\left(R_3^2+z^2\right)^{3/2}} + \frac{\mu_0 i R_3}{4\pi\left(z^2+R_3^2\right)}\right]\hat{k}}$$

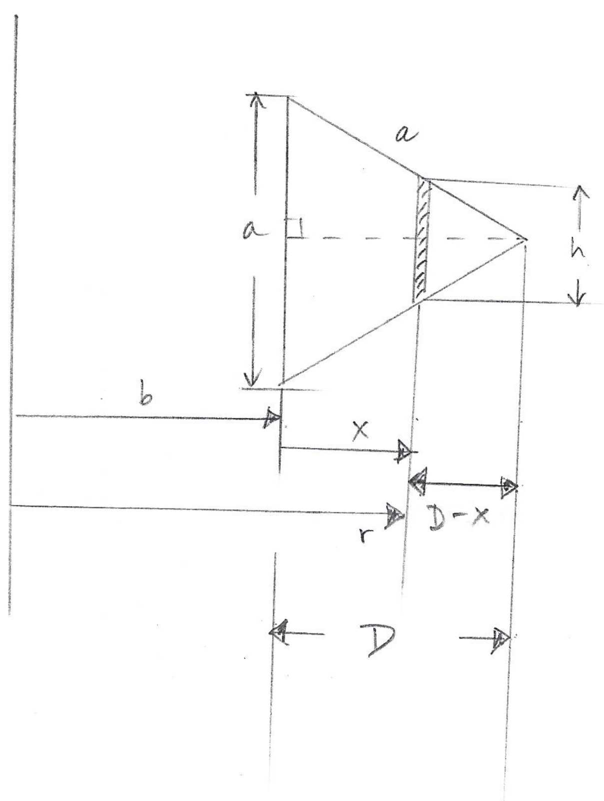
#2:



$$\oint_l \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$B 2\pi r = \mu_0 i N$$

$$\therefore\ B = \frac{\mu_0 i N}{2\pi r} \quad \text{—— (1)}$$



$$D = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

$$\therefore\ D = \frac{\sqrt{3}}{2}a$$

also, $b + x = r$

$$x = r - b$$

$$\Phi_B = \int \vec{B}\cdot d\vec{A} \longleftarrow (1)$$

$$\Phi_B = \int \frac{\mu_0 i N}{2\pi r}(h\cdot dr)$$

similar Δ's:

$$\frac{h}{a} = \frac{D - x}{D}$$

$$D = \frac{\sqrt{3}}{2}a$$

$$x = r - b$$

$$\therefore\ \frac{h}{\cancel{a}} = \frac{\frac{\sqrt{3}}{2}a - r + b}{\frac{\sqrt{3}}{2}\cancel{a}}$$

$$\therefore\ h = a + \frac{2}{\sqrt{3}}b - \frac{2}{\sqrt{3}}r$$

$$\therefore \quad \Phi_B = \int \frac{\mu_0 i N}{2\pi r}\left(a + \frac{2}{\sqrt{3}}b - \frac{2}{\sqrt{3}}r\right)\cdot dr$$

$$= \frac{\mu_0 i N}{2\pi}\int_{r=b}^{b+\frac{\sqrt{3}}{2}a}\left(\frac{a+\frac{2}{\sqrt{3}}b}{r} - \frac{2}{\sqrt{3}}\right)dr$$

$$= \frac{\mu_0 i N}{2\pi}\left[\left[\left(a+\frac{2}{\sqrt{3}}b\right)\ln\left|b+\frac{\sqrt{3}}{2}a\right| - \frac{2}{\sqrt{3}}\left(b+\frac{\sqrt{3}}{2}a\right)\right] - \left[\left(a+\frac{2}{\sqrt{3}}\right)\ln|b| - \frac{2}{\sqrt{3}}b\right]\right]$$

$$= \frac{\mu_0 i N}{2\pi}\left[\left(a+\frac{2}{\sqrt{3}}b\right)\ln\left|\frac{b+\frac{\sqrt{3}}{2}a}{b}\right| + \frac{2}{\sqrt{3}}\left(\cancel{b} - \cancel{b} - \frac{\sqrt{3}}{2}a\right)\right]$$

$$= \frac{\mu_0 i N}{2\pi}\left[\left(a+\frac{2}{\sqrt{3}}b\right)\ln\left|\frac{b+\frac{\sqrt{3}}{2}a}{b}\right| - a\right]$$

$$\Phi_B = \frac{\mu_0 i N}{2\pi}\left(a+\frac{2}{\sqrt{3}}b\right)\left[\ln\left|\frac{b+\frac{\sqrt{3}}{2}a}{b}\right| - \frac{a}{a+\frac{2}{\sqrt{3}}b}\right]$$

$$Li = \Phi_T$$

$$\Phi_T = N\cdot\Phi_B$$

$$\therefore \quad L = \frac{N\Phi_B}{i}$$

$$L = \frac{\mu_0 N^2}{2\pi}\left(a+\frac{2}{\sqrt{3}}b\right)\left[\ln\left|\frac{b+\frac{\sqrt{3}}{2}a}{b}\right| - \frac{a}{a+\frac{2}{\sqrt{3}}b}\right] \quad \text{—— } 2$$

for res. ν.

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi\nu$$

$$\therefore \quad \sqrt{LC} = \frac{1}{2\pi\upsilon}$$

$$C = \frac{1}{L}\left(\frac{1}{2\pi\upsilon}\right)^2$$

$$C = \frac{1}{4\pi^2 L \upsilon^2} \longleftarrow 2$$

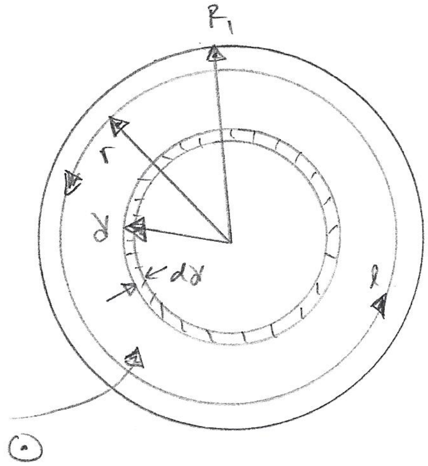
$$C = \frac{\cancel{2\pi}}{2\cancel{4}\pi^{\cancel{2}}\mu_0 N^2\left(a+\frac{2}{\sqrt{3}}b\right)\left[\ln\left|\frac{b+\frac{\sqrt{3}}{2}a}{b}\right| - \frac{a}{a+\frac{2}{\sqrt{3}}b}\right]\cdot\upsilon^2}$$

$$C = \frac{1}{2\pi\mu_0(200)^2\left(5m+\frac{2}{\sqrt{3}}(20m)\right)\left[\ln\left|\frac{20m+\frac{\sqrt{3}}{2}(5m)}{20m}\right| - \frac{5m}{5m+\frac{2}{\sqrt{3}}(20m)}\right]\left(386.0077983\times10^{-3}Hz\right)^2}$$

$$\therefore \quad \boxed{C = 42F}$$

#3:

a) $B(r<R_1)$:



$3i$ ⊙

$j \propto (4\gamma^n - 5\gamma^{n+1})$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$B\,2\pi r = \mu_0 \cdot \int_0^r di$$

$$di = j\,dA$$

$$j = k(4\gamma^n - 5\gamma^{n+1})$$

$$dA = 2\pi\gamma\, d\gamma$$

$$di = 2\pi k\,(4\gamma^{n+1} - 5\gamma^{n+2})\,d\gamma$$

$$\int_0^{R_1} di = 3i = 2\pi k\left(\frac{4R_1^{n+2}}{n+2} - \frac{5R_1^{n+3}}{n+3}\right)$$

$$\therefore\ k = \frac{3i}{2\pi\left(\frac{4R_1^{n+2}}{n+2} - \frac{5R_1^{n+3}}{n+3}\right)}$$

$$\int_0^r di = 2\pi k\left(\frac{4r^{n+2}}{n+2} - \frac{5r^{n+3}}{n+3}\right)$$

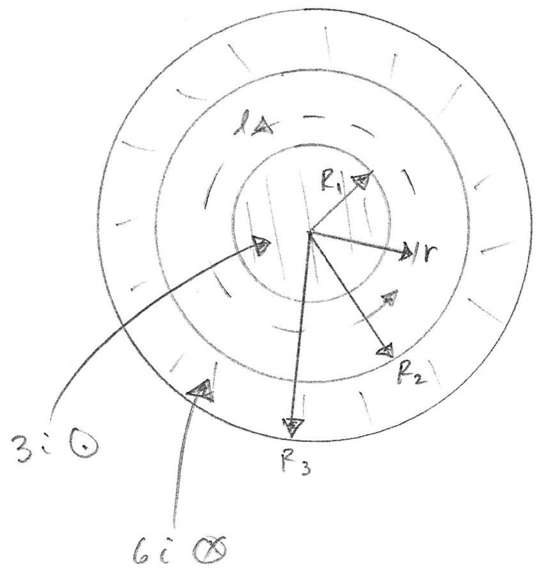
$$\therefore\ B\,2\pi r = \mu_0\left[\cancel{2\pi}\left(\frac{3i}{\cancel{2\pi}\left(\frac{4R_1^{n+2}}{n+2} - \frac{5R_1^{n+3}}{n+3}\right)}\right)\left(\frac{4r^{n+2}}{n+2} - \frac{5r^{n+3}}{n+3}\right)\right]$$

$$\therefore\ B = \frac{3\mu_0 i}{2\pi r}\cdot\left[\frac{\frac{4r^{n+2}}{n+2} - \frac{5r^{n+3}}{n+3}}{\frac{4R_1^{n+2}}{n+2} - \frac{5R_1^{n+3}}{n+3}}\right]$$

$$= \frac{3\mu_0 i}{2\pi r}\left[\frac{4(n+3)r^{n+2} - 5(n+2)r^{n+3}}{\cancel{(n+2)}(n+3)}\right]\left[\frac{\cancel{(n+2)}(n+3)}{4(n+3)R_1^{n+2} - 5(n+2)R_1^{n+3}}\right]$$

$$\boxed{B = \frac{3\mu_0 i}{2\pi r}\left[\frac{4(n+3)r^{n+2} - 5(n+2)r^{n+3}}{4(n+3)R_1^{n+2} - 5(n+2)R_1^{n+3}}\right]}$$

b) $B(R_1 < r < R_2)$:

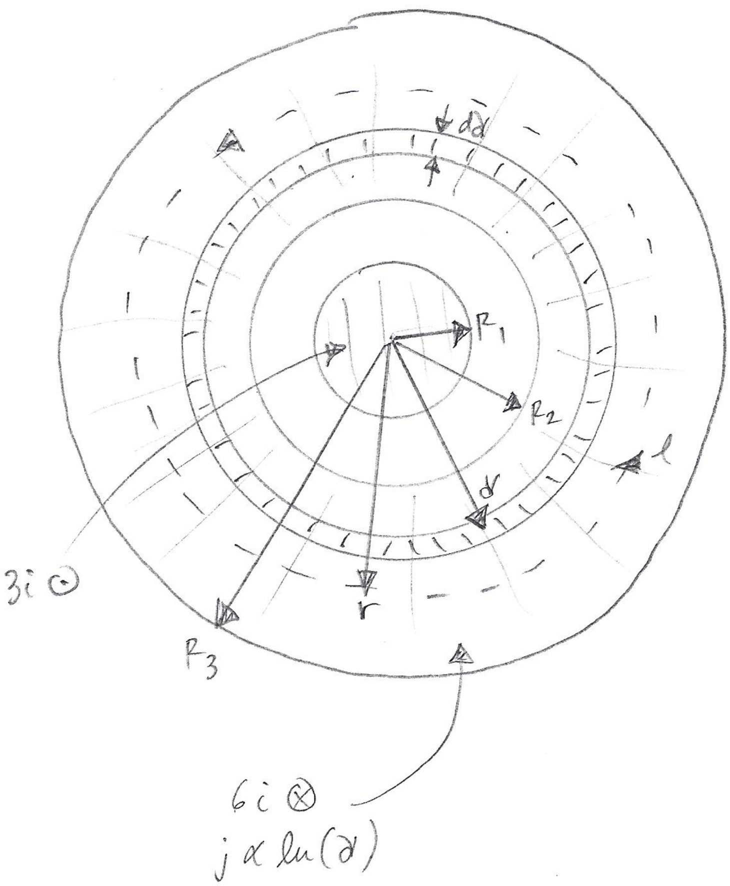


$$\oint_l \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$B\,2\pi r = \mu_0(3i)$$

$$\boxed{B = \frac{3\mu_0 i}{2\pi r}}$$

c) $B(R_2 < r < R_3)$:



$$\oint_l \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$B\,2\pi r = \mu_0\left(3i - \int_{R_2}^{r} di\right)$$

$$di = j\,dA$$

$$j = k\ln(\vartheta)$$

$$dA = 2\pi\vartheta\,d\vartheta$$

$$di = 2\pi k\,\vartheta\ln(\vartheta)\,d\vartheta$$

$$\int_{R_2}^{R_3} di = 6i = 2\pi k\int_{R_2}^{R_3}\vartheta\ln(\vartheta)\,d\vartheta$$

let $u = \ln\vartheta$ $\quad dv = \vartheta\,d\vartheta$

$du = \frac{1}{\vartheta}d\vartheta$ $\quad v = \frac{1}{2}\vartheta^2$

by parts:

$$6i = 2\pi k\left[\tfrac{1}{2}\vartheta^2\ln\vartheta - \int\tfrac{1}{2}\vartheta^2\cdot\tfrac{1}{\vartheta}\,d\vartheta\right]_{R_2}^{R_3}$$

$$6i = 2\pi k\left[\tfrac{1}{2}\vartheta^2\ln\vartheta - \tfrac{1}{4}\vartheta^2\right]_{R_2}^{R_3}$$

$$6i = 2\pi k\,\tfrac{1}{4}\vartheta^2(2\ln\vartheta - 1)\Big|_{R_2}^{R_3}$$

$$\downarrow$$

$$\therefore \quad 6i = \frac{\pi k}{2}\left[R_3^2(2\ln R_3 - 1) - R_2^2(2\ln R_2 - 1)\right]$$

$$k = \frac{12i}{\pi\left(R_3^2(2\ln R_3 - 1) - R_2^2(2\ln R_2 - 1)\right)}$$

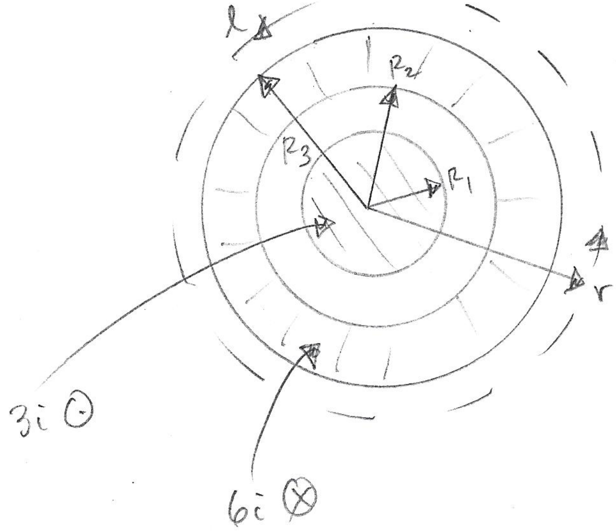
$$\int_{R_2}^{r} di = 2\pi k \frac{1}{4}\gamma^2(2\ln\gamma - 1)\Big|_{R_2}^{r}$$

$$= \frac{\cancel{\pi}}{\cancel{2}}\left(\frac{\cancelto{6}{12} i}{\cancel{\pi}\left[R_3^2(2\ln R_3 - 1) - R_2^2(2\ln R_2 - 1)\right]}\right)\left[r^2(2\ln r - 1) - R_2^2(2\ln R_2 - 1)\right]$$

$$\therefore \quad B 2\pi r = \mu_0\left[3i - 6i \cdot \frac{r^2(2\ln r - 1) - R_2^2(2\ln R_2 - 1)}{R_3^2(2\ln R_3 - 1) - R_2^2(2\ln R_2 - 1)}\right]$$

$$\boxed{B = \frac{3\mu_0 i}{2\pi r}\left[1 - 2 \cdot \frac{r^2(2\ln r - 1) - R_2^2(2\ln R_2 - 1)}{R_3^2(2\ln R_3 - 1) - R_2^2(2\ln R_2 - 1)}\right]}$$

d) $B(r > R_3)$:



$$\oint_l \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$B 2\pi r = \mu_0(3i - 6i)$$

$$\therefore \quad \boxed{B = -\frac{3\mu_0 i}{2\pi r}}$$

e) $\frac{dB}{dr}\left(r = \frac{R_1}{4}\right) = 0$ for max B @ $R_1/4$

Inside first cylinder:

$$B(r) = \frac{3\mu_0 i}{2\pi r}\left[\frac{4(n+3)r^{n+2} - 5(n+2)r^{n+3}}{4(n+3)R_1^{n+2} - 5(n+2)R_1^{n+3}}\right]$$

$$B(r) = \cancel{\frac{3\mu_0 i}{2\pi\left[4(n+3)R_1^{n+2} - 5(n+2)R_1^{n+3}\right]}}^{K} \cdot \left(4(n+3)r^{n+1} - 5(n+2)r^{n+2}\right)$$

$$\therefore B(r) = K\left(4(n+3)r^{n+1} - 5(n+2)r^{n+2}\right)$$

$$\frac{dB}{dt} = K\left(4(n+3)(n+1)r^{n} - 5(n+2)(n+2)r^{n+1}\right) = 0$$ @ $R_1/4$ for max B.

$$\therefore \quad r = R_1/4$$

so $\cancel{K}\left[4(n+3)(n+1)\left(\frac{R_1}{4}\right)^{n} - 5(n+2)(n+2)\left(\frac{R_1}{4}\right)^{n+1}\right] = 0$

$$\cancel{\left(\frac{R_1}{4}\right)^{n}}\left[4(n+3)(n+1) - 5(n+2)(n+2)\left(\frac{R_1}{4}\right)\right] = 0$$

$$\therefore 4(n^2+4n+3) - \frac{5}{4}R_1(n^2+4n+4) = 0$$

$$16(n^2+4n+3) - 5R_1(n^2+4n+4) = 0$$

$$16n^2 - 5R_1n^2 + 64n - 20R_1n + 48 - 20R_1 = 0$$

$$\therefore (16-5R_1)n^2 + (64-20R_1)n + (48-20R_1) = 0$$

$$n = \frac{-(64-20R_1) \pm \sqrt{(64-20R_1)^2 - 4(16-5R_1)(48-20R_1)}}{2(16-5R_1)}$$

$$n = \frac{-4(16-5R_1) \pm \sqrt{16(16-5R_1)^2 - 16(16-5R_1)(12-5R_1)}}{2(16-5R_1)}$$

$$n = -2 \pm \frac{4\sqrt{(16-5R_1)((16-5R_1)-(12-5R_1))}}{2(16-5R_1)}$$

$$n = -2 \pm 2 \cdot \frac{\sqrt{16-5R_1}}{16-5R_1} \cdot \cancelto{2}{\sqrt{16-12-\cancel{5R_1}+\cancel{5R_1}}}$$

$$\therefore \quad n = -2 \pm 4\,\frac{\sqrt{16-5R_1}}{16-5R_1}$$

$$n = -2 \pm \frac{4}{\sqrt{16-5R_1}}$$

If we take $n > 0$

$$\boxed{n = 2\left(\frac{2}{\sqrt{16-5R_1}} - 1\right)}$$

$$R_1 < \frac{16}{5} \; ; \quad \frac{2}{\sqrt{16-5R_1}} > 1 \qquad \therefore \quad \underline{\underline{\frac{12}{5} < R_1 < \frac{16}{5}}}$$

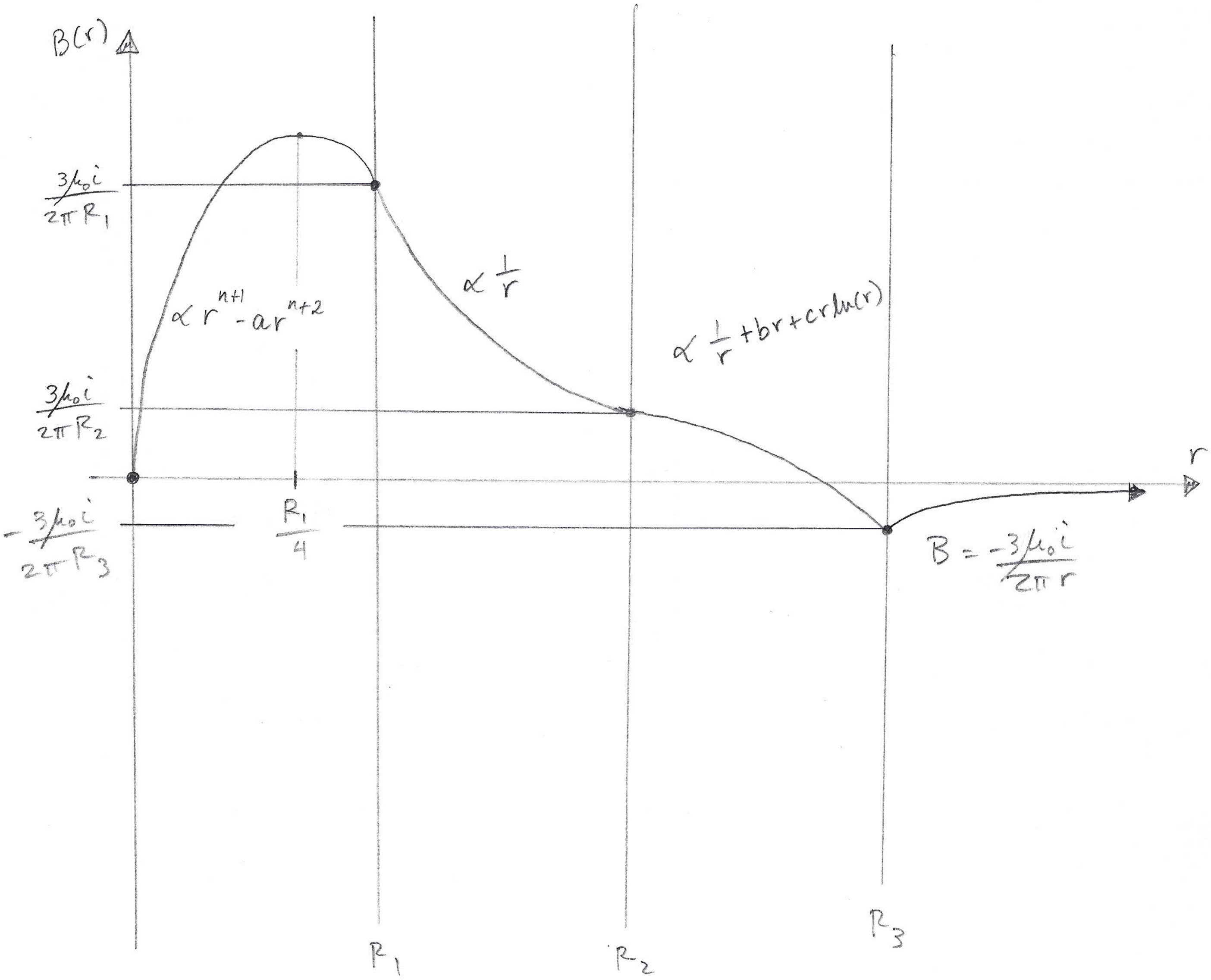
$$2 > \sqrt{16-5R_1}$$

$$4 > 16 - 5R_1$$

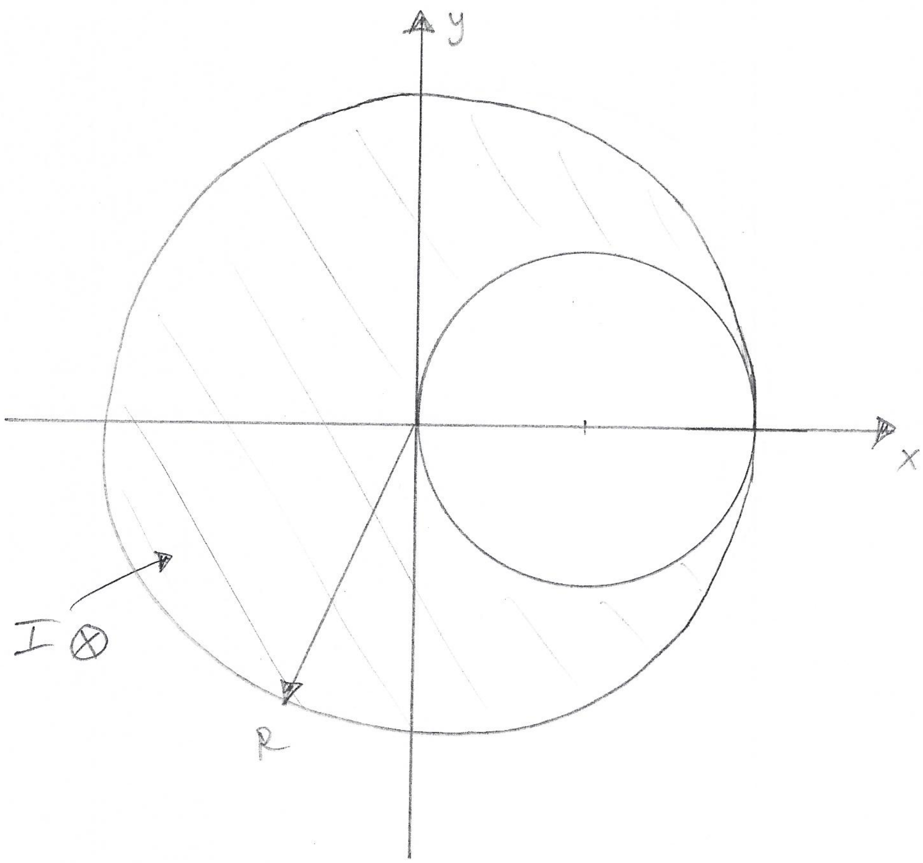
$$5R_1 > 12$$

$$R_1 > \frac{12}{5}$$

f)


B(r)
r
$\frac{3\mu_0 i}{2\pi R_1}$
$\frac{3\mu_0 i}{2\pi R_2}$
$-\frac{3\mu_0 i}{2\pi R_3}$
$\frac{R_1}{4}$
$\propto r^{n+1} - a r^{n+2}$
$\propto \frac{1}{r}$
$\propto \frac{1}{r} + br + cr\ln(r)$
$B = \frac{-3\mu_0 i}{2\pi r}$
R_1
R_2
R_3

#4:



To find the magnetic field at points inside the wire, we can apply the principle of superposition.

That is, we will find the magnetic field due to a solid cylinder of radius R with current $I_1 \otimes$. We then find the magnetic field due to a solid cylinder of radius $R/2$ with current $I_2 \odot$ at the same position of interest. We then add the magnetic fields vectorally.

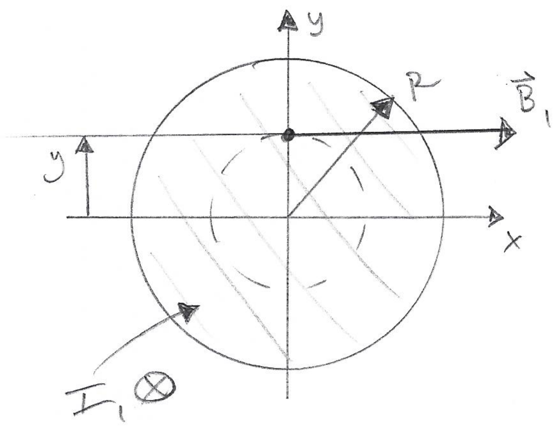
We are essentially "subtracting out" the hollow section.

$$\therefore \quad \vec{B}_T = \vec{B}_1 + \vec{B}_2$$

cylinder of radius R

cylinder of radius R/2

Cylinder 1:



$$\oint \vec{B}_1 \cdot d\vec{s} = \mu_0 i_{enc}$$

$$B_1 2\pi y = \mu_0 i_{enc} \quad \text{—— (1)}$$

$$i_{enc} = j \cdot A_{enc}$$

$$j = \frac{I_1}{\pi R^2}$$

$$A_{enc} = \pi y^2$$

$$\therefore \quad i_{enc} = \frac{y^2}{R^2} I_1 \quad \text{—— (2)}$$

To find I_1, we can use a ratio:

$$\frac{I_1}{A_1} = \frac{I}{\underbrace{A_1 - A_2}_{}}$$

solid section of our initial wire with current I.

$$\frac{I_1}{\pi R^2} = \frac{I}{\pi R^2 - \pi (R/2)^2}$$

$$\frac{I_1}{\cancel{R^2}} = \frac{I}{\frac{3}{4}\cancel{R^2}}$$

$$\therefore \quad I_1 = \frac{4}{3} I \quad \text{—— (3)}$$

③ → ② → ①

$$B_1 \cancel{2\pi y} = \mu_0 \left(\frac{y^{\cancel{2}}}{R^2} \left(\frac{\cancel{4}^{\,2}}{3} I \right) \right)$$

$$\therefore \; B_1 = \frac{2 \mu_0 I y}{3 \pi R^2}$$

$$\vec{B}_1 = \frac{2 \mu_0 I y}{3 \pi R^2} \, \hat{i} \;\text{———}\; ④$$

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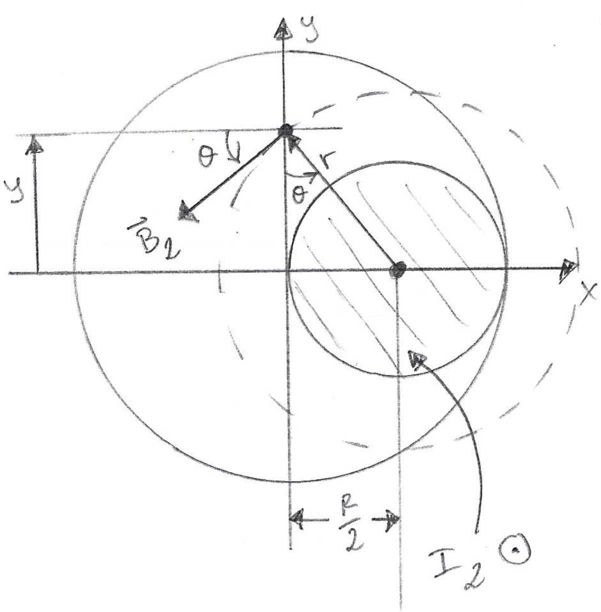
cylinder 2:



$$\oint \vec{B}_2 \cdot d\vec{s} = \mu_0 i_{enc}$$

$$B_2 2\pi r = \mu_0 i_{enc}$$

$i_{enc} = I_2$ for all y

$$r = \sqrt{y^2 + \left(\tfrac{R}{2}\right)^2}$$

$$r = \tfrac{1}{2}\sqrt{4y^2 + R^2}$$

$$\therefore \; B_2 \cancel{2}\pi \cdot \tfrac{1}{\cancel{2}} \sqrt{4y^2 + R^2} = \mu_0 I_2 \;\text{———}\; ⑤$$

$$\frac{I_2}{A_2} = \frac{I}{A_1 - A_2}$$

$$\frac{I_2}{\cancel{\pi}\left(\frac{R}{2}\right)^2} = \frac{I}{\cancel{\pi} R^2 - \cancel{\pi}\left(\frac{R}{2}\right)^2}$$

$$\frac{I_2}{R^2/\cancel{4}} = \frac{I}{3R^2/\cancel{4}}$$

$$\therefore \; I_2 = \tfrac{1}{3} I \longrightarrow ⑤$$

$$\therefore \; B_2 = \frac{\frac{1}{3}\mu_0 I}{\pi\sqrt{4y^2+R^2}}$$

$$\vec{B}_2 = -\frac{\mu_0 I}{3\pi\sqrt{4y^2+R^2}} \cdot \cos\theta \, \hat{i} - \frac{\mu_0 I}{3\pi\sqrt{4y^2+R^2}} \cdot \sin\theta \, \hat{j}$$

$$\cos\theta = \frac{y}{\sqrt{y^2+(R/2)^2}} \qquad \sin\theta = \frac{R/2}{\sqrt{y^2+(R/2)^2}}$$

$$\therefore \cos\theta = \frac{y}{\frac{1}{2}\sqrt{4y^2+R^2}} \qquad \sin\theta = \frac{R}{\sqrt{4y^2+R^2}}$$

$$\therefore \quad \vec{B}_2 = -\frac{2\mu_0 I y}{3\pi(4y^2+R^2)}\hat{i} - \frac{\mu_0 I R}{3\pi(4y^2+R^2)}\hat{j} \quad \text{——— } 6$$

$$\vec{B}_T = \vec{B}_1 + \vec{B}_2 \longleftarrow 4 \; \& \; 6$$

$$\vec{B}_T = \left(\frac{2\mu_0 I y}{3\pi R^2} - \frac{2\mu_0 I y}{3\pi(4y^2+R^2)}\right)\hat{i} - \frac{\mu_0 I R}{3\pi(4y^2+R^2)}\hat{j}$$

$$\vec{B}_T = \frac{2\mu_0 I y}{3\pi}\left(\frac{1}{R^2} - \frac{1}{4y^2+R^2}\right)\hat{i} - \frac{\mu_0 I R}{3\pi(4y^2+R^2)}\hat{j}$$

$$\vec{B}_T = \frac{2\mu_0 I y}{3\pi}\left(\frac{4y^2+\cancel{R^2}-\cancel{R^2}}{R^2(4y^2+R^2)}\right)\hat{i} - \frac{\mu_0 I R}{3\pi(4y^2+R^2)}\hat{j}$$

$$\therefore \quad \vec{B}_T = \frac{8\mu_0 I y^3}{3\pi R^2(4y^2+R^2)}\hat{i} - \frac{\mu_0 I R}{3\pi(4y^2+R^2)}\hat{j}$$

$$\|\vec{B}_T\| = \frac{\mu_0 I}{3\pi(4y^2+R^2)}\sqrt{\left(\frac{8y^3}{R^2}\right)^2 + (-R)^2}$$

$$\|\vec{B}_T\| = \frac{\mu_0 I}{3\pi(4y^2+R^2)} \sqrt{\frac{64y^6}{R^4} + R^2}$$

$$= \frac{\mu_0 I}{3\pi(4y^2+R^2)} \sqrt{\frac{64y^6 + R^6}{R^4}}$$

$$\boxed{\|\vec{B}_T\| = \frac{\mu_0 I \sqrt{64y^6 + R^6}}{3\pi R^2(4y^2+R^2)}}$$

$$B(0) = \frac{\mu_0 I}{3\pi R^2} \cdot \frac{\sqrt{0 + R^6}}{(0+R^2)}$$

$$= \frac{\mu_0 I R^3}{3\pi R^4}$$

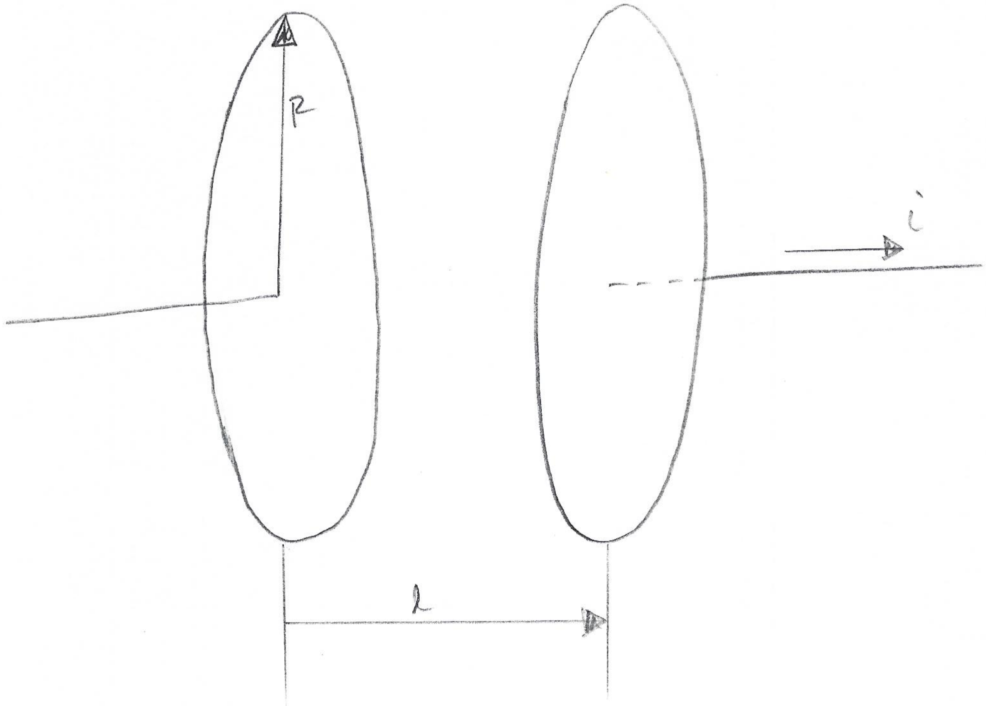
$$\boxed{B(0) = \frac{\mu_0 I}{3\pi R}}$$

$$B(R) = \frac{\mu_0 I}{3\pi R^2} \frac{\sqrt{64R^6 + R^6}}{(4R^2+R^2)}$$

$$B(R) = \frac{\mu_0 I \sqrt{65} \cdot R^3}{3\pi R^2 (5R^2)}$$

$$\boxed{B(R) = \frac{\sqrt{65}\, \mu_0 I}{15\pi R}}$$

\#5 :



$V = 800\,8135\ V$

$R = 42\ km$

$l_0 = 1\,cm$

$\frac{dl}{dt} = 4\,cm/s$

a) Since cap is fully charged @ $t=0$, there is an initial charge Q_0 on the plates.

$Q = CV$ @ any pt. in time.

* V is constant always (constant V source).

$C = \frac{A\varepsilon_0}{l}$ for a parallel plate cap.

l is changing

$\therefore$ c is changing

$Q = CV$

$\therefore$ Q is changing!

So by separating the plates, we change the capacitance and cause a current to flow in order to change the charge on the plates.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

To maintain a continuous flow within the circuit

$$i = i_{dis} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$i = \varepsilon_0 \frac{d}{dt}(E\cdot A)$$

$$\int \vec{E}\cdot\vec{dl} = V$$

$$E\int dl = El = V$$

$$\therefore \quad E = V/l$$

$$A = \pi R^2$$

$$\therefore \quad i = \varepsilon_0 \frac{d}{dt}\left(\frac{V\pi R^2}{l}\right)$$

V & R are constant!

$$i = \varepsilon_0 V \pi R^2 \frac{d}{dt}\left(\frac{1}{l}\right)$$

$$i = \varepsilon_0 V \pi R^2 \left(-\frac{1}{l^2}\frac{dl}{dt}\right)$$

we only care about magnitude though

$$\therefore \quad i = \frac{\varepsilon_0 V \pi R^2}{l^2}\frac{dl}{dt} \quad \text{———— (1)}$$

$$l(t) = \int \frac{dl}{dt}\, dt$$

$\hookrightarrow$ constant!

$$l(t) = \frac{dl}{dt}\int dt$$

$$\therefore \quad l(t) = \frac{dl}{dt}\cdot t + l_0 \quad \text{———— (2)}$$

② → ①

$$\boxed{i(t) = \frac{\varepsilon_0 V \pi R^2 \frac{dl}{dt}}{\left(\frac{dl}{dt}\cdot t + l_0\right)^2}}$$

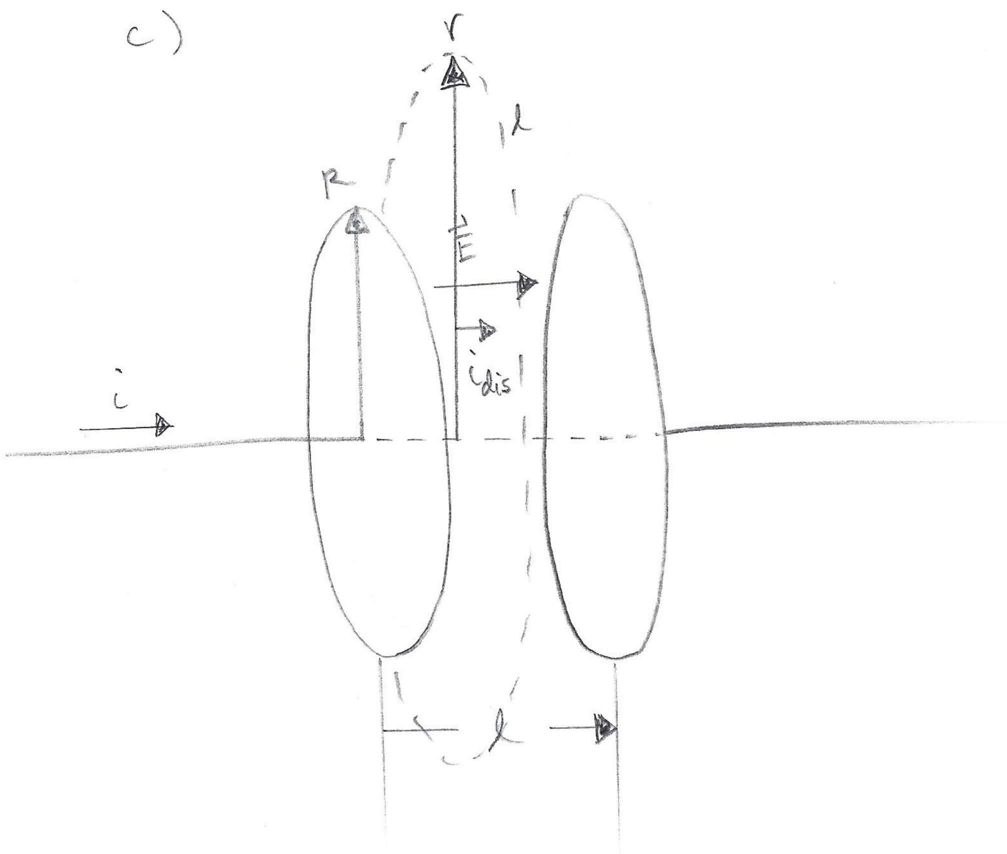
b) ① $i = \dfrac{\varepsilon_0 V \pi R^2 \frac{dl}{dt}}{l^2}$

$$\therefore \; l = \sqrt{\frac{\varepsilon_0 V \pi R^2 \frac{dl}{dt}}{i}}$$

$$l = \sqrt{\frac{(8.85\times10^{-12}\,\tfrac{F}{m})(8008135\,V)\pi(42\times10^{-3}\,m)^2(0.04\,m/s)}{(174558.3354\,A)}}$$

$$\boxed{l = 30 \text{ cm}}$$

c)



$r = 43\ km$

$\varepsilon_{rms} = 2500\ V$

$\omega = 69\ rad/s$

$l = 0.118167522 5\ nm$

$$B_{rms} = \frac{B_{max}}{\sqrt{2}}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d}{dt}(E \cdot A) \quad \varepsilon(t)$$

$$B 2\pi r = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{\cancel{V}\, \pi R^2}{l}\right)$$

Radius of the area of the plates

radius of our amperian loop to find B.

$$B 2\pi r = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\underbrace{\frac{\pi R^2}{l}}_{\text{constant}} \varepsilon_{max} \sin \omega t\right)$$

$$B 2\pi r = \mu_0 \varepsilon_0 \frac{\pi R^2 \varepsilon_{max}}{l} \cdot \omega \cos \omega t$$

$$\varepsilon_{max} = \sqrt{2} \cdot \varepsilon_{rms}$$

$$B 2\pi r = \frac{\mu_0 \varepsilon_0 \pi R^2 \mathcal{E}_{rms} \cdot \sqrt{2}\, \omega \cos\omega t}{l}$$

for B_{max}, $\cos\omega t = 1$!

$$\therefore \quad B_{max} = \frac{\sqrt{2}\mu_0 \varepsilon_0 \mathcal{E}_{rms} R^2 \omega}{2 l r}$$

$$\therefore \quad B_{rms} = \frac{\mu_0 \varepsilon_0 \mathcal{E}_{rms} R^2 \omega}{2 l r}$$

$$= \frac{(4\pi \times 10^{-7} \tfrac{Tm}{A})(8.85\times 10^{-12} \tfrac{F}{m})(2500V)(42\times 10^{3} m)^2 (69 \text{ rad/s})}{2(0.118167 5225 \times 10^{-9} m)(43\times 10^{3} m)}$$

$$\boxed{B_{rms} = 333 \text{ T}}$$

d)

$$i_{dis} = \varepsilon_0 \frac{d}{dt}(E \cdot A)$$

$$= \varepsilon_0 \frac{d}{dt}\left(\frac{V \pi R^2}{l}\right)$$

$$V = \mathcal{E}(t) = \mathcal{E}_{max} \sin\omega t$$

$$= \varepsilon_0 \frac{d}{dt}\left(\frac{\pi R^2}{l} \mathcal{E}_{max} \sin\omega t\right)$$

$$i_{dis} = \frac{\varepsilon_0 \pi R^2 \mathcal{E}_{max} \cdot \omega \cos\omega t}{l}$$

$$\mathcal{E}_{max} = \sqrt{2} \cdot \mathcal{E}_{rms}$$

max i_{dis} occurs when $\cos\omega t = 1$

$$\therefore \quad i_{dis_{max}} = \frac{\sqrt{2}\, \varepsilon_0 \mathcal{E}_{rms} \pi R^2 \omega}{l}$$

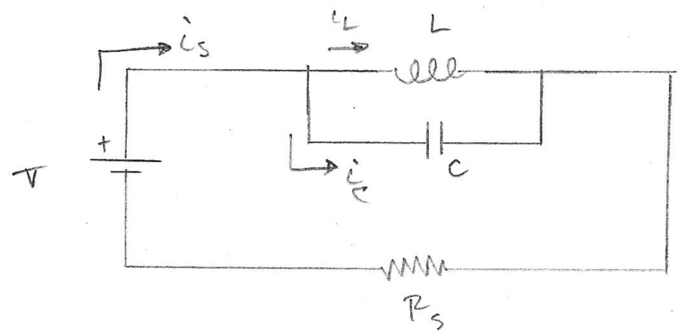
$$i_{dis_{max}} = \frac{\sqrt{2}\left(8.85\times10^{-12}\,\frac{F}{m}\right)(2500V)\,\pi\left(42\times10^{3}m\right)^2\left(69\,\frac{rad}{s}\right)}{\left(0.1181675225\times10^{-9}m\right)}$$

$$\boxed{i_{dis_{max}} = 1.0125062\times10^{14}\,A}$$

Be careful, 10^{14} A is sort of a lot of current.

#6:

a) S_1 & S_2 closed:



$V = 25\ V$
$R = 15\ \Omega$
$L = 60\ mH$
$C = 12\ \mu F$
$R_s = 20\ \Omega$

@ $t = \infty$, $\left.\begin{array}{l} i_L = i_s \\ i_c = 0 \end{array}\right\}$ DC!

$$E_L = \frac{1}{2} L i_L^2$$

$$i_L = i_s = \frac{V}{R_s}$$

$$\therefore\ i_L = \frac{25\ V}{20\ \Omega} = 1.25\ A$$

$$E_L = \frac{1}{2}(60\times10^{-3}\ H)(1.25\ A)^2$$

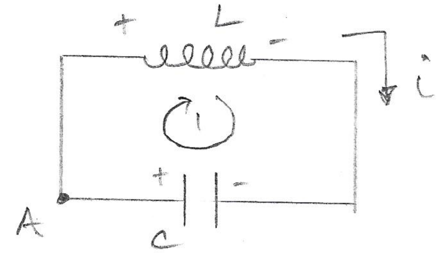
$$\boxed{E_L = 0.046875\ J}$$

b) $$E_c = \frac{1}{2} C V_c^2$$

$$V_c = V_L = L\,\cancel{\frac{di}{dt}} \rightarrow 0 \text{ (DC)}$$

$$\therefore\ \boxed{E_c = 0\ J}$$

c) Open S_1 :



<u>loop 1 @ A :</u>

$$-V_L + V_c = 0$$

$$V_L = -L\frac{di}{dt}$$

negative since the magnetic field will <u>collapse</u> to produce current when the source is removed.

$$V_c = q/c$$

$$\therefore \quad L\frac{di}{dt} + \frac{1}{c}q = 0$$

$$\ddot{q} + \frac{1}{LC}q = 0 \leftarrow \text{Simple Harmonic}$$

let $q = q_{max}\sin(\omega t + \phi)$

@ $t=0$, S_1 is opened.

Since $V_L = 0$ @ $t=0$ (DC)

our $V_c = 0$ @ $t=0$ too

$\therefore \quad q(0) = \cancel{CV} 0$

$$0 = q_{max}\sin(\omega \cancel{t}^{0} + \phi)$$

$$\therefore \quad \phi = 0$$

$$\dot{q} = q_{max}\, \omega \cos \omega t$$

$$\ddot{q} = -q_{max}\, \omega^2 \sin \omega t$$

$$\ddot{q} + \frac{1}{LC} q = 0$$

$$-\cancel{q_{max}}\, \omega^2 \cancel{\sin \omega t} + \frac{1}{LC} \cancel{q_{max}} \cancel{\sin \omega t} = 0$$

$$\therefore \omega = \sqrt{\frac{1}{LC}}$$

$$i(t) = \dot{q} = q_{max} \overset{\sqrt{\frac{1}{LC}}}{\cancel{\omega}} \cos \omega t$$

$$\therefore i(t) = \frac{q_{max}}{\sqrt{LC}} \cos \omega t \longrightarrow (1)$$

we need q_{max}.

$E_T = E_c + E_L$ at any time.

$$E_T = \frac{1}{2} C V^2 + \frac{1}{2} L i^2$$

@ $t = 0$ $V_c = 0$

$$E_T = E_L\,(t=0) = 0.046875 \text{ J}$$

when q is max, $E_T = E_c = 0.046875$ J

$$\therefore \frac{1}{2} C V^2 = E_T$$

$$V = q_{max}/C$$

$$\therefore \frac{1}{2} \frac{q_{max}^2}{C} = E_T$$

$$q_{max} = \sqrt{2 C E_T} \longrightarrow (1)$$

$$\therefore \quad i_L(t) = i_c(t) = \frac{\sqrt{2CE_T}}{\sqrt{LC}} \cos\omega t = i(t)$$

$$i(t) = \sqrt{\frac{2\cancel{C}E_T}{L\cancel{C}}} \cos\omega t$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \boxed{i(t) = \sqrt{\frac{2E_T}{L}} \cos\left[\frac{1}{\sqrt{LC}} t\right]}$$

where $E_T = 0.046875\ J$

d) $V(t) = V_c(t) = V_L(t)$

$$V_L = -L \frac{di}{dt}$$

$$= -L \frac{d}{dt}\left(\sqrt{\frac{2E_T}{L}} \cos\left(\frac{1}{\sqrt{LC}} t\right)\right)$$

$$= -L \cdot \sqrt{\frac{2E_T}{L}} \left(-\frac{1}{\sqrt{LC}} \sin\left(\frac{1}{\sqrt{LC}} t\right)\right)$$

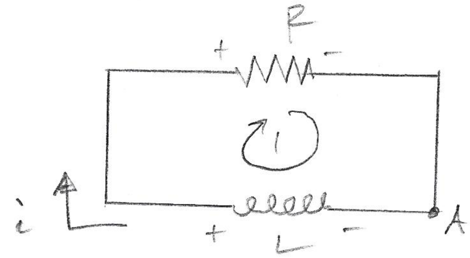
$$\therefore \quad V_L = \frac{\sqrt{2L^2E_T}}{\sqrt{L^2 C}} \sin\left(\frac{1}{\sqrt{LC}} t\right)$$

$$V_L = \sqrt{\frac{2E_T}{C}} \sin\left(\frac{1}{\sqrt{LC}} t\right)$$

$$\therefore \quad \boxed{V(t) = \sqrt{\frac{2E_T}{C}} \sin\left(\frac{1}{\sqrt{LC}} t\right)}$$

$E_T = 0.046875\ J$

e) S_2 is opened, S_3 is closed.
* when cap is empty!



$$E_{diss} = \int_0^\infty i^2 R\,dt \quad \text{———} \quad (2)$$

find i!

<u>loop 1 @ A:</u>

$$+V_L - V_R = 0$$

$$V_L = -L\frac{di}{dt} \quad \text{(collapsing B field)}$$

$$V_R = iR$$

$$\therefore \quad L\frac{di}{dt} + iR = 0$$

$$\int_{i_0}^{i} \frac{di}{i} = -\int_0^t \frac{R}{L}\,dt$$

$$\ln\left|\frac{i}{i_0}\right| = -\frac{R}{L}t$$

$$i = i_0 e^{-R/L \cdot t} \quad \text{———} \quad (3)$$

@ $t=0, \; E_c = 0$

$\therefore \; E_L = E_T = 0.046875 \text{ J}$

$\frac{1}{2} L i_0^2 = E_T$

$i_0 = \sqrt{\frac{2E_T}{L}} \longrightarrow 3$

$\therefore \quad i = \sqrt{\frac{2E_T}{L}} \, e^{-\frac{R}{L}t} \longrightarrow 2$

$$E_{diss} = \int_0^{\infty} R \cdot \left(\sqrt{\frac{2E_T}{L}} \, e^{-\frac{R}{L}t} \right)^2 dt$$

$$= \frac{2E_T R}{L} \int_0^{\infty} e^{-\frac{2R}{L}t} \, dt$$

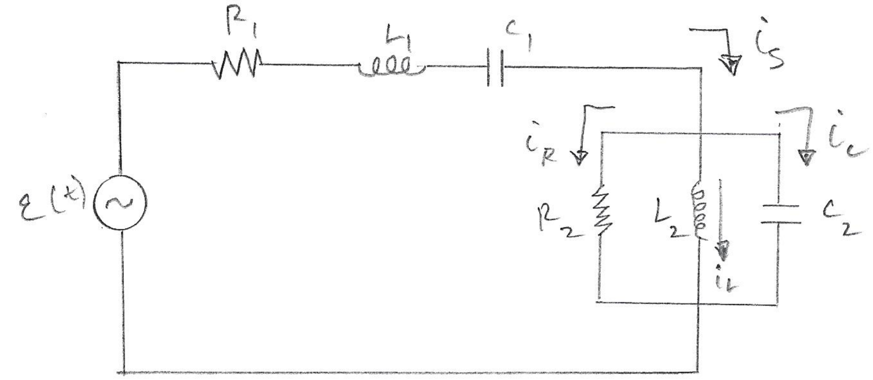
$$= \frac{\cancel{2} E_T \cancel{R}}{\cancel{L}} \left(-\frac{\cancel{L}}{\cancel{2}\cancel{R}} e^{-\frac{2R}{L}t} \right) \Big|_0^{\infty}$$

$$= E_T \left(e^0 - \cancel{e^{\infty}} \right) \rightarrow 1$$

$\therefore \; \boxed{E_{diss} = E_T = 0.046875 \text{ J}}$

cons. of E!

#7 :



$\mathcal{E}_{rms} = 120\ V$

$\nu = 60\ Hz$

$\therefore\ \omega = 120\pi\ \frac{rad}{s}$

$R_1 = 30\ \Omega$

$R_2 = 25\ \Omega$

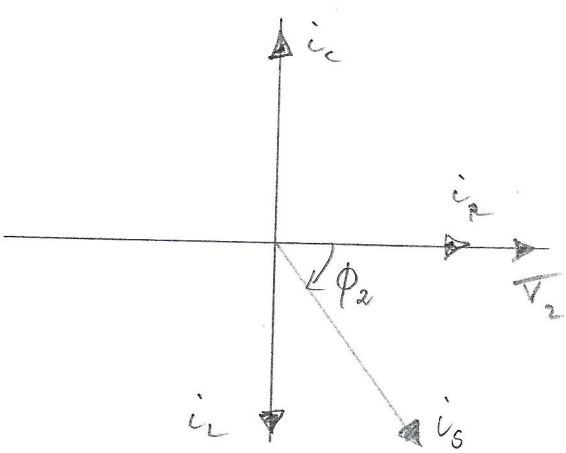
$L_1 = 17\ mH$

$L_2 = 15\ mH$

$C_1 = 500\ \mu F$

$C_2 = 300\ \mu F$

a) Parallel Branch:



$$\vec{i}_S = \vec{i}_R + \vec{i}_L + \vec{i}_C$$

$$i_S = \sqrt{(i_C - i_L)^2 + i_R^{\,2}}$$

$$\frac{V_2}{Z_2} = \sqrt{\left(\frac{V_2}{X_{C_2}} - \frac{V_2}{X_{L_2}}\right)^2 + \left(\frac{V_2}{R}\right)^2}$$

$$\therefore\ Z_2 = \frac{1}{\sqrt{\left(\omega C_2 - \frac{1}{\omega L_2}\right)^2 + \left(\frac{1}{R_2}\right)^2}}$$

$$\therefore\ Z_2 = \frac{1}{\sqrt{\left((120\pi)(300\times10^{-6}) - \frac{1}{(120\pi)(15\times10^{-3})}\right)^2 + \left(\frac{1}{25}\right)^2}}$$

$Z_2 = 13.28854953\ \Omega$ ——— (1)

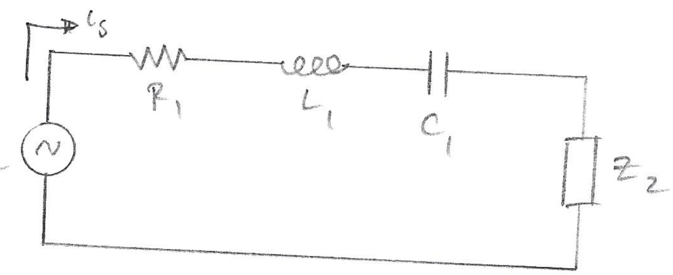
$$\phi_2 = \arctan\left(\frac{i_c - i_L}{i_R}\right)$$

$$= \arctan\left[\frac{\dfrac{\cancel{V_2}}{X_{c_2}} - \dfrac{\cancel{V_2}}{X_{L_2}}}{\dfrac{\cancel{V_2}}{R_2}}\right]$$

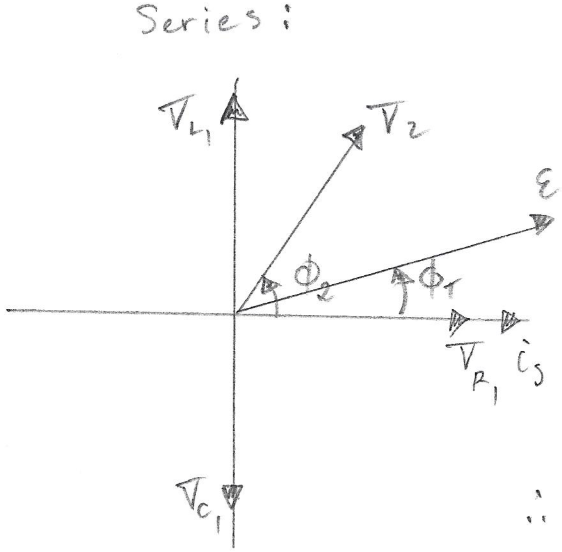
$$\therefore \quad \phi_2 = \arctan\left(\frac{\omega C_2 - \frac{1}{\omega L_2}}{1/R_2}\right)$$

$$= \arctan\left(\frac{(120\pi)(300\times10^{-6}) - \frac{1}{(120\pi)(15\times10^{-3})}}{\left(\frac{1}{25}\right)}\right)$$

$$\phi_2 = 57.89030039^\circ \text{ ——— } (2)$$



Series:



$$\vec{\varepsilon} = \vec{V}_{R_1} + \vec{V}_{L_1} + \vec{V}_{C_1} + \vec{V}_2$$

$$\varepsilon = \sqrt{(V_{R_1} + V_2\cos\phi_2)^2 + (V_{L_1} + V_2\sin\phi_2 - V_{C_1})^2}$$

$$i_s Z_T = \sqrt{(i_s R_1 + i_s Z_2\cos\phi_2)^2 + (i_s X_{L_1} + i_s Z_2\sin\phi_2 - i_s X_{C_1})^2}$$

$$\therefore \quad Z_T = \sqrt{(R_1 + Z_2\cos\phi_2)^2 + (X_{L_1} - X_{C_1} + Z_2\sin\phi_2)^2}$$

①② $\longrightarrow$ $$Z_T = \sqrt{\left((30+(13.288\cdots)\cos(57.89\cdots)\right)^2 + \left((120\pi)(17\times10^{-3}) - \frac{1}{(120\pi)(500\times10^{-6})} + (13.28\cdots)\sin(57.89\cdots)\right)^2}$$

$$\boxed{Z_T = 39.06986998\ \Omega}$$

b) $$\phi_T = \arctan\left(\frac{V_{L_1} - V_{C_1} + V_2 \sin\phi_2}{V_{R_1} + V_2\cos\phi_2}\right)$$

$$= \arctan\left(\frac{i_s X_{L_1} - i_s X_{C_1} + i_s Z_2 \sin\phi_2}{i_s R_1 + i_s Z_2 \cos\phi_2}\right)$$

$$\phi_T = \arctan\left(\frac{\omega L_1 - \frac{1}{\omega C_1} + Z_2 \sin\phi_2}{R_1 + Z_2\cos\phi_2}\right) \longleftarrow ①②$$

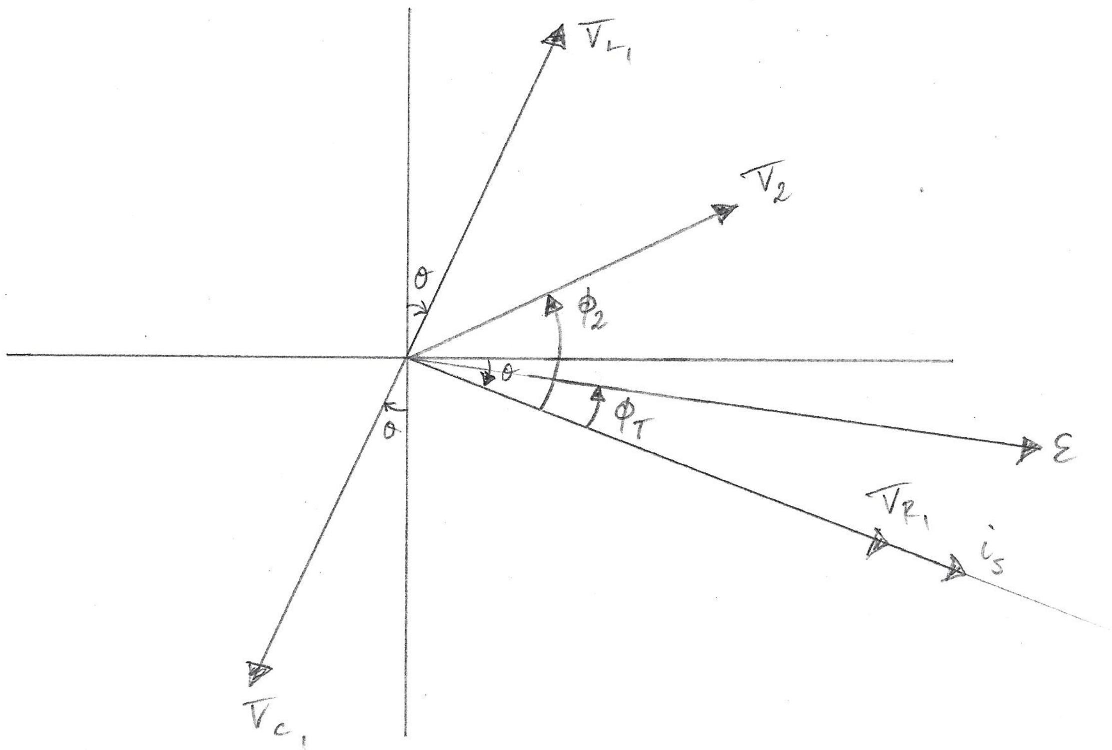
$$\phi_T = 18.44195547^\circ$$

$\therefore$ $\boxed{\varepsilon \text{ leads } i \text{ by } 18.44195547^\circ}$

c) $i(t) = -0.5\, \cancel{i_{max}} = \cancel{i_{max}} \sin\theta$

$\therefore \quad \theta = \arcsin(-\tfrac{1}{2})$

$\theta = -30^\circ$



$V_{R_1} = V_{R_{1max}} \sin\theta$

$V_{R_{1max}} = i_{s_{max}} R_1$

$i_{s_{max}} = \dfrac{\cancel{\mathcal{E}_{max}} \to \sqrt{2}\,\mathcal{E}_{rms}}{Z_T}$

$\therefore \quad V_{R_1} = \dfrac{\sqrt{2}\,\mathcal{E}_{rms} R_1}{Z_T} \sin\theta$

$V_{R_1} = \dfrac{\sqrt{2}\,(120V)(30\Omega)}{39.069\ldots\Omega} \sin(-30^\circ)$

$\boxed{V_{R_1} = -65.15466813\ V}$

$$V_{L_1} = V_{L_1 max} \cos|\theta|$$

$$= i_{s_{max}} X_{L_1} \cos|\theta|$$

$$= \frac{\sqrt{2}\,\mathcal{E}_{rms}}{Z_T} \cdot \omega L_1 \cos|\theta|$$

$$V_{L_1} = \frac{\sqrt{2}\,(120V)(120\pi \cdot 17\times10^{-3}\,\Omega)}{(39.0\ldots\Omega)} \cos 30^\circ$$

$$\boxed{V_{L_1} = 24.10820911\ V}$$

$$V_{C_1} = -V_{C_1 max} \cos|\theta|$$

$$= -i_{s_{max}} X_{C_1} \cos|\theta|$$

$$= -\frac{\sqrt{2}\,\mathcal{E}_{rms}}{Z_T \omega C_1} \cos|\theta|$$

$$V_{C_1} = -\frac{\sqrt{2}\,(120V)\left(\frac{1}{(120\pi)(500\times10^{-6})}\Omega\right)}{(39.0\ldots\Omega)} \cos 30^\circ$$

$$\boxed{V_{C_1} = -19.9564729\ V}$$

$$V_{R_2} = V_{L_2} = V_{C_2} = V_2$$

$$V_2 = V_{2_{max}} \sin(\phi_2 - |\theta|)$$

$$= \frac{\sqrt{2}\,\mathcal{E}_{rms}}{Z_T} Z_2 \cdot \sin(\phi_2 - |\theta|)$$

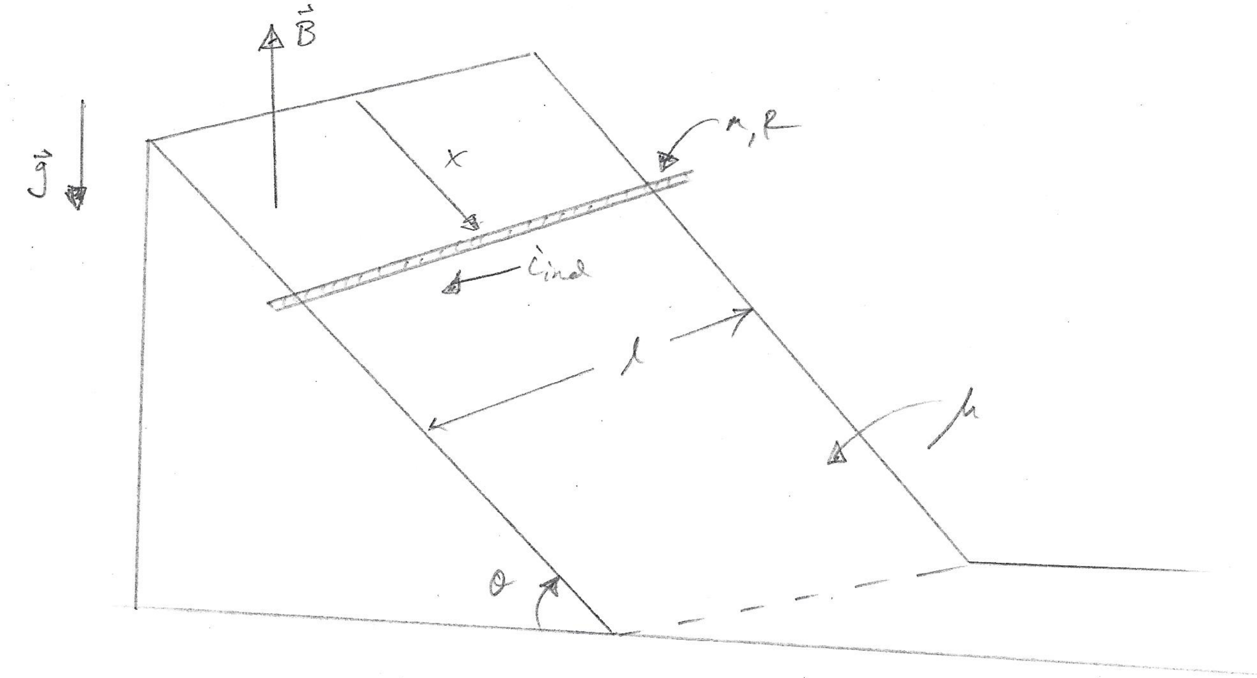
$$\boxed{V_2 = 27.00061696\ V}$$

$$\mathcal{E} = -\sqrt{2}\,\mathcal{E}_{rms} \cdot \sin(|\theta| - \phi_T)$$

$$\boxed{\mathcal{E} = -34.00231496\ V}$$

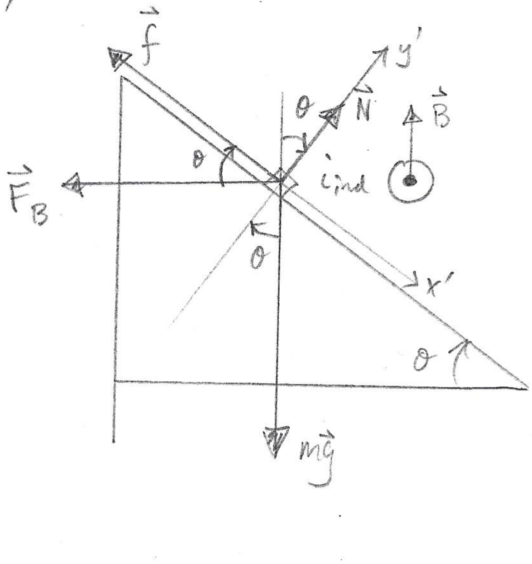
$$-65.15 + 24.11 - 19.96 + 27.00 = -34.00 \checkmark$$

#8:



a) Φ_B increasing, $\therefore$ i_{ind} $\boxed{\text{clockwise}}$ to decrease $\vec{B}$.

b)



$\underline{x'}$

$$mg\sin\theta - F_B\cos\theta - f = m\ddot{x} \quad \text{—— (1)}$$

$\underline{y'}$

$$N - mg\cos\theta - F_B \cdot \sin\theta = 0 \quad \text{—— (2)}$$

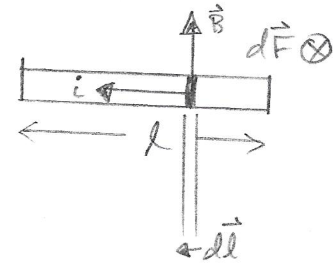
$$f = \mu \cdot N \quad \text{—— (3)}$$

(2) $N - mg\cos\theta - F_B\sin\theta = 0$

$N = mg\cos\theta + F_B\sin\theta \longrightarrow (3) \longrightarrow (1)$

$\therefore \quad mg\sin\theta - F_B\cos\theta - \left(\mu\left[mg\cos\theta + F_B\sin\theta\right]\right) = m\ddot{x}$

$\therefore \quad m\ddot{x} = -F_B(\cos\theta + \mu\sin\theta) + mg(\sin\theta - \mu\cos\theta) \quad \text{———} \quad (1')$



$\vec{F}_B = q\vec{v}\times\vec{B}$

$d\vec{F}_B = dq\,\vec{v}\times\vec{B}$

$= dq\,\dfrac{d\vec{l}}{dt}\times\vec{B}$

$= i\,d\vec{l}\times\vec{B}$

$\therefore \quad dF_B = iB\,dl$

$\therefore \quad F_B = ilB$

$i = i_{ind} = \dfrac{\varepsilon_{ind}}{R}$

$\varepsilon_{ind} = \left|\dfrac{d\Phi_B}{dt}\right|$

$= \left|\dfrac{d}{dt}\int \vec{B}\cdot d\vec{A}\right|$

$= \left|\dfrac{d}{dt}\int B\,dA\cos\theta\right|$

$\varepsilon_{ind} = B\cos\theta\left|\dfrac{d}{dt}A\right|$

$A = l\cdot x$

$\therefore \quad \varepsilon_{ind} = Bl\cos\theta\,\dot{x}$

$F_B = \left[\dfrac{(Bl\cos\theta\,\dot{x})}{R}\right]\cdot lB$

$F_B = \dfrac{B^2 l^2\cos\theta}{R}\dot{x} \quad \text{———} \quad (4)$

(4) $\rightarrow$ (1')

$$m\ddot{x} = -\frac{B^2 l^2 \cos\theta}{R}(\cos\theta + \mu\sin\theta)\dot{x} + mg(\sin\theta - \mu\cos\theta)$$

$$\frac{dv}{dt} = -\frac{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}{mR} v + g(\sin\theta - \mu\cos\theta)$$

$$\frac{dv}{dt} = -\frac{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}{mR}\left[v - \frac{mRg(\sin\theta - \mu\cos\theta)}{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}\right]$$

let $K = -\dfrac{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}{mR}$

and $\eta = -\dfrac{mRg(\sin\theta - \mu\cos\theta)}{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}$

$$\therefore \quad \frac{dv}{dt} = K(v+\eta)$$

$$\int \frac{dv}{v+\eta} = \int k\,dt$$

$$\ln|v+\eta| \Big|_{\cancel{v_0}\,0}^{v(t)} = kt \Big|_0^t$$

$$\ln\left|\frac{v+\eta}{\eta}\right| = kt$$

$$\frac{v+\eta}{\eta} = e^{kt}$$

$$v = \eta e^{kt} - \eta$$

$$v = \eta(e^{kt} - 1)$$

$$X(t) = \int v\,dt$$

$$= \int \eta\,(e^{kt} - 1)\,dt$$

$$x(t) = \frac{\eta}{k} e^{kt} - \eta t + x_0 \quad \leftarrow \eta, k$$

$$\therefore\ x(t) = \frac{\left[-\frac{mRg(\sin\theta - \mu\cos\theta)}{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}\right]}{\left[-\frac{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}{mR}\right]} e^{-\frac{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}{mR} t}$$

$$- \left[-\frac{mRg(\sin\theta - \mu\cos\theta)}{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}\right] \cdot t + X_0$$

$$\therefore\ \boxed{x(t) = X_0 + \frac{mRg(\sin\theta - \mu\cos\theta)}{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}\left[t + \frac{mR \cdot e^{-\frac{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}{mR} t}}{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}\right]}$$

c) $E_{diss} = E_R + |W_f|$ ——— (5)

$$E_R = \int i^2 R\, dt$$

$$i = \frac{\varepsilon_{ind}}{R} = \frac{Bl\cos\theta}{R} \cdot \dot{x}$$

$$\dot{x} = v = \eta\,(e^{kt} - 1)$$

$$\therefore E_R = R\left(\frac{Bl\cos\theta}{R}\right)^2 \cdot \int (\eta\,(e^{kt}-1))^2 dt$$

$$E_R = \frac{B^2 l^2 \cos^2\theta}{R} \cdot \eta^2 \int_{t=0}^{t} (e^{2kt} - 2e^{kt} + 1)\, dt$$

$$E_R = \frac{B^2 l^2 \cos^2\theta}{R} \eta^2 \left(\frac{1}{2k} e^{2kt} - \frac{2}{k} e^{kt} + t\right)$$ ——— (6)

$$|W_f| = \left| \int_{x=x_0}^{x} \vec{f} \cdot d\vec{x} \right|$$

$$= \left| -\int_{x_0}^{x} f\, dx \right|$$

$$= \left| \int_{x_0}^{x} \mu\,(mg\cos\theta + F_B \sin\theta)\, dx \right|$$

$$F_B = \frac{B^2 l^2 \cos\theta}{R} \dot{x}$$

$$= \mu \left| \int_{x_0}^{x} (mg\cos\theta + \frac{B^2 l^2 \cos\theta \sin\theta}{R} \dot{x}\, dx \right|$$

$$= \mu\cos\theta \left| \int_{x_0}^{x} mg\, dx + \frac{B^2 l^2 \sin\theta}{R} \int \dot{x}\, dx \right|$$

$$= \mu mg\cos\theta(x-x_0) + \frac{\mu\cos\theta\sin\theta B^2 l^2}{R}\left|\int v\,dx\right|$$

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$\therefore |W_f| = \mu mg\cos\theta(x-x_0) + \frac{\mu B^2 l^2\cos\theta\sin\theta}{R}\int(\dot{x})^2\,dt$$

$$\dot{x} = \eta(e^{kt}-1)$$

$$= \mu mg\cos\theta(x-x_0) + \frac{\mu B^2 l^2\cos\theta\sin\theta}{R}\int \eta^2(e^{2kt}-2e^{kt}+1)\,dt$$

$$|W_f| = \mu mg\cos\theta(x-x_0) + \frac{\mu B^2 l^2\cos\theta\sin\theta\,\eta^2}{R}\left(\frac{1}{2k}e^{2kt}-\frac{2}{k}e^{kt}+t\right) \quad (7)$$

$$\therefore E_{diss} = E_R + |W_f| \leftarrow (6)\,(7)$$

$$= \frac{B^2 l^2\cos^2\theta\,\eta^2}{R}\left(\frac{1}{2k}e^{2kt}-\frac{2}{k}e^{kt}+t\right) + \mu mg\cos\theta(x-x_0)$$

$$+ \frac{\mu B^2 l^2\cos\theta\sin\theta\,\eta^2}{R}\left(\frac{1}{2k}e^{2kt}-\frac{2}{k}e^{kt}+t\right)$$

$$E_{diss} = \mu mg\cos\theta(x-x_0) + \frac{B^2 l^2\cos\theta\,\eta^2}{R}\left(\frac{1}{2k}e^{2kt}-\frac{2}{k}e^{kt}+t\right)\left[\cos\theta+\mu\sin\theta\right]$$

$x(t)$, η, k

$$E_{diss} = \mu m g \cancel{\cos\theta} \left[\frac{mRg(\sin\theta - \mu\cos\theta)}{B^2 l^2 \cancel{\cos\theta}(\cos\theta + \mu\sin\theta)} \right] \Bigg[t + \frac{mR\, e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t}}{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)} \Bigg]$$

$$+ \frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{R} \left[-\frac{mRg(\sin\theta-\mu\cos\theta)}{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)} \right]^2 \Bigg[- \frac{mR\, e^{-2\cdot\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t}}{2B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}$$

$$+ \frac{2mR\, e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t}}{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)} + t \Bigg]$$

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$$E_{diss} = \frac{\mu m^2 g^2 R(\sin\theta - \mu\cos\theta)}{B^2 l^2 (\cos\theta+\mu\sin\theta)} \Bigg[t + \frac{mR}{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)} e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t} \Bigg]$$

$$+ \frac{\cancel{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}}{\cancel{R}} \cdot \frac{m^2 R^2 g^2 (\sin\theta-\mu\cos\theta)^2}{B^4 l^4 \cos^2\theta(\cos\theta+\mu\sin\theta)^2} \cdot \frac{m\cancel{R}}{\cancel{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}} \Bigg[\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t$$

$$+ 2e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t} - \frac{1}{2} e^{-2\cdot\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t} \Bigg]$$

$$\frac{\mu m^3 g^2 R^2 (\sin\theta - \mu\cos\theta)}{B^4 l^4 (\cos\theta + \mu\sin\theta)^2}$$

$$= \cancel{\frac{\mu m^2 g^2 R (\sin\theta - \mu\cos\theta)}{B^2 l^2 (\cos\theta + \mu\sin\theta)} \cdot \frac{mR}{B^2 l^2 \cos\theta(\cos\theta + \mu\sin\theta)}} \left[e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t} + \frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t \right]$$

$$+ \frac{m^3 R^2 g^2 (\sin\theta - \mu\cos\theta)^2}{B^4 l^4 \cos^2\theta (\cos\theta + \mu\sin\theta)^2} \left[\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t + 2e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t} - \frac{1}{2} e^{-2\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t} \right]$$

$$= \frac{m^3 g^2 R^2 (\sin\theta - \mu\cos\theta)}{B^4 l^4 (\cos\theta + \mu\sin\theta)^2} \left[\mu e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t} + \frac{\mu B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR} t \right.$$

$$+ \frac{(\sin\theta - \mu\cos\theta)}{\cos^{\cancel{2}}\theta} \cdot \frac{B^2 l^2 \cancel{\cos\theta}(\cos\theta+\mu\sin\theta)}{mR} t + \frac{2(\sin\theta - \mu\cos\theta)}{\cos^2\theta} e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t}$$

$$\left. - \frac{1}{2} \frac{(\sin\theta - \mu\cos\theta)}{\cos^2\theta} e^{-\frac{2B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t} \right]$$

$$\boxed{\begin{aligned} E_{diss} = \frac{m^3 g^2 R^2 (\sin\theta - \mu\cos\theta)}{B^4 l^4 (\cos\theta + \mu\sin\theta)^2} \Bigg[& \frac{B^2 l^2 (\cos\theta + \mu\sin\theta)}{mR} \left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta} + \mu\cos\theta \right) \cdot t \\ & + \left(\mu + \frac{2(\sin\theta - \mu\cos\theta)}{\cos^2\theta} \right) e^{-\frac{B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t} \\ & - \frac{(\sin\theta - \mu\cos\theta)}{2\cos^2\theta} e^{-\frac{2B^2 l^2 \cos\theta(\cos\theta+\mu\sin\theta)}{mR}t} \Bigg] \end{aligned}}$$